

# Fourier-Optics Compatible Radiation Propagation Methods Used in SRW



O. Chubar, NSLS-II, BNL

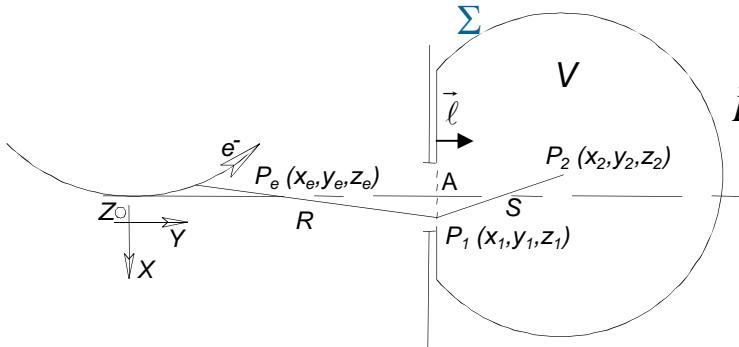
Collaboration Meeting on "Simulation and Modeling for SR Sources and X-Ray Optics"

October 1 - 2, 2015, NSLS-II



# Wavefront Propagation in the Case of Full Transverse Coherence

**Kirchhoff Integral Theorem** applied to Spontaneous Emission by One Electron



$$\vec{E}_{\omega 2\perp}(P_2) \approx \frac{k^2 e}{4\pi} \int_{-\infty}^{+\infty} d\tau \iint_A \frac{\vec{\beta}_{e\perp} - \vec{n}_\perp}{RS} \exp[ik(c\tau + R + S)] \cdot (\vec{l} \cdot \vec{n}_{p_e p_1} + \vec{l} \cdot \vec{n}_{p_1 p_2}) d\Sigma$$

Valid at large observation angles;  
Is applicable to complicated cases of diffraction inside vacuum chamber

**Huygens-Fresnel Principle**

$$\vec{E}_{\omega 2\perp}(P_2) \approx \frac{k}{4\pi i} \iint_A \vec{E}_{\omega 1\perp}(P_1) \frac{\exp(ikS)}{S} (\vec{l} \cdot \tilde{\vec{n}} + \vec{l} \cdot \vec{n}_{p_1 p_2}) d\Sigma$$

**Fourier Optics**

**Free Space:**  
(between parallel planes perpendicular to optical axis)

$$\vec{E}_{\omega 2\perp}(x_2, y_2) \approx \frac{k}{2\pi i L} \iint \vec{E}_{\omega 1\perp}(x_1, y_1) \exp[ik[L^2 + (x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}] dx_1 dy_1$$

Assumption of small angles

**"Thin"** Optical Element:

$$\vec{E}_{\omega 2\perp}(x, y) \approx \mathbf{T}(x, y, \omega) \vec{E}_{\omega 1\perp}(x, y)$$

**"Thick"** Optical Element:  
(propagation from transverse plane before the element to a transverse plane just after it)

$$\vec{E}_{\omega 2\perp}(x_2, y_2) \approx \mathbf{G}(x_2, y_2, \omega) \exp[ik\Lambda(x_2, y_2, k)] \vec{E}_{\omega 1\perp}(x_1(x_2, y_2), y_1(x_2, y_2))$$

Implemented in SRW for Python in 2012;

Currently used for simulation of NSLS-II PX and spectral microscopy beamlines

Benchmarking against experimental data is required

# Approach to High-Accuracy Partially-Coherent Emission and Wavefront Propagation Simulations

**Averaging** (over phase-space volume occupied by e-beam) of the **intensity** (or mutual intensity, or mathematical brightness) obtained from **electric field emitted by an electron** and **propagated** through an optical system:

$$\begin{aligned} I_{\omega}(x, y) &= \int I_{\omega l}(x, y; x_e, y_e, z_e, x'_e, y'_e, \delta\gamma_e) f(x_e, y_e, z_e, x'_e, y'_e, \delta\gamma_e) dx_e dy_e dz_e dx'_e dy'_e d\delta\gamma_e \\ I_{\omega l}(x, y; x_e, y_e, z_e, x'_e, y'_e, \delta\gamma_e) &= |\mathbf{E}_{\omega l\perp}(x, y; x_e, y_e, z_e, x'_e, y'_e, \delta\gamma_e)|^2 \\ M_{\omega l}(x, y, \tilde{x}, \tilde{y}; x_e, y_e, z_e, x'_e, y'_e, \delta\gamma_e) &= \mathbf{E}_{\omega l\perp}(x, y; x_e, y_e, z_e, x'_e, y'_e, \delta\gamma_e) \mathbf{E}_{\omega l\perp}^*(\tilde{x}, \tilde{y}; x_e, y_e, z_e, x'_e, y'_e, \delta\gamma_e) \\ B_{\omega l}(x, y, \theta_x, \theta_y; x_e, y_e, z_e, x'_e, y'_e, \delta\gamma_e) &\sim \mathbf{E}_{\omega l\perp}(x, y; x_e, y_e, z_e, x'_e, y'_e, \delta\gamma_e) \int \mathbf{E}_{\omega l\perp}^*(\tilde{x}, \tilde{y}; x_e, y_e, z_e, x'_e, y'_e, \delta\gamma_e) \exp\left[i \frac{\omega}{c} (\theta_x \tilde{x} + \theta_y \tilde{y})\right] d\tilde{x} d\tilde{y} \end{aligned}$$

This method is **general** and **accurate**. For the most part, it is already implemented in SRW code. However, it can be **CPU-intensive**, requiring **parallel calculations** on a multi-core server or a small cluster. Several approaches are considered for increasing the efficiency, including use of low-discrepancy sequences (collaboration with R. Lindberg, K.-J. Kim, X. Shi, ANL), "improved Monte-Carlo" type techniques, as well as "coherent mode decomposition".

NOTE: the **smaller** the **e-beam emittance** (the higher the radiation coherence) – the **faster** is the **convergence** of simulations with this general method.

NOTE: **convolution** can be valid in some cases, such as pure projection geometry, focusing by a thin lens, diffraction at one slit, etc.

$$I_{\omega}(x, y) \approx \int \tilde{I}_{\omega l}(x - \tilde{x}_e, y - \tilde{y}_e) \tilde{f}(\tilde{x}_e, \tilde{y}_e) d\tilde{x}_e d\tilde{y}_e$$

If convolution is valid, the **calculations can be accelerated** dramatically. The validity of the convolution relation can be easily verified numerically.

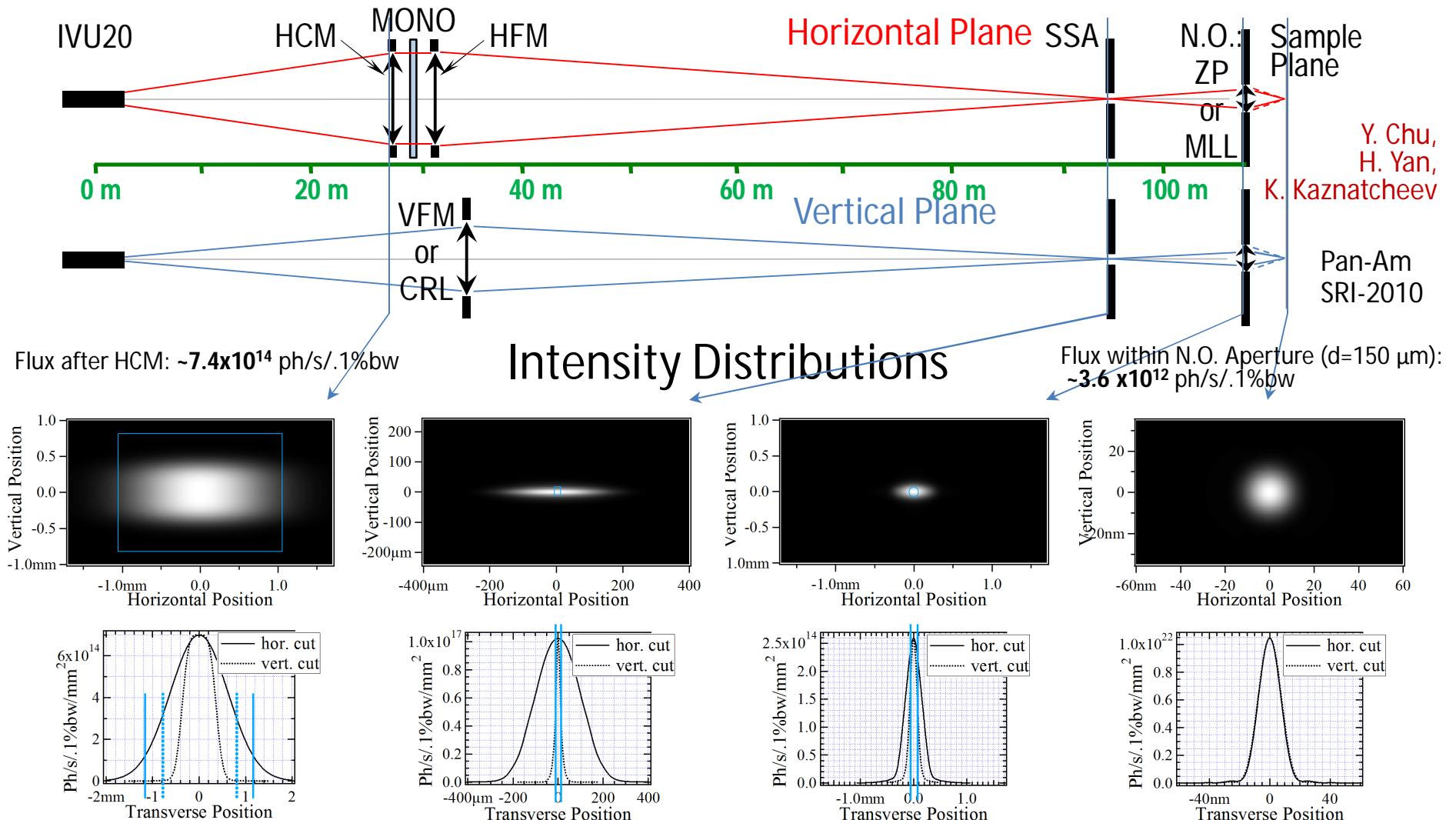
# Updates of Core SRW Functions

## Made at NSLS-II (in collaboration with other Labs)

- Accurate partially-coherent emission and wavefront propagation simulations for SR sources are possible with SRW since ~2009:  
O.Chubar, Y.S.Chu, K.Kaznatcheev, H.Yan, AIP Conf. Proc. Vol. 1234, pp.75-78 (2009)  
O.Chubar, Y.S.Chu, K.Kaznatcheev, H.Yan, Nucl. Instr. and Meth., vol. A649, Issue 1, pp.118-122 (2011)
- Parallel calculations of Partially-Coherent Emission and Wavefront Propagation are implemented in SRW for Python (based on MPI / mpi4py). Besides "normal" Intensity, calculation of Mutual Intensity / Degree of Coherence is possible:  
O.Chubar, A.Fluerasu, L.Berman, K.Kaznatcheev, L.Wiegart, J. Phys.: Conf. Ser. 425, 162001 (2013)  
D.Laundy, J.P.Sutter, U.H.Wagner, C.Rau, C.A.Thomas, K.J.S.Sawhney, and O.Chubar, J. Phys.: Conf. Ser. 425, 162002 (2013)
- Increased reliability of Time- / Frequency-Dependent FEL Pulse Propagation simulations:  
S.Roling, H.Zacharias, L.Samoylova, H.Sinn, Th.Tschentscher, O.Chubar, A.Buzmakov, E.Schneidmiller, M.V.Yurkov, F.Siewert, S.Braun, and P.Gawlitz, Phys. Rev. ST Accel. Beams 17, 110705 (2014)
- New physical-optics "propagators" are implemented for:
  - Grazing-Incidence Focusing Mirrors, using the stationary phase method / "local ray-tracing":  
N.Canestrari, O.Chubar, R.Reininger, J. Synchrotron Rad. 21, 1110-1121 (2014)
  - Perfect Crystals, using the X-ray Dynamical Diffraction methods:  
J.P.Sutter, O.Chubar, A.Suvorov, Proc. SPIE Vol. 9209, 92090L (2014)  
A.Suvorov, Y.Q.Cai, J.P.Sutter, O.Chubar, Proc. SPIE Vol. 9209, 92090H (2014)
  - Variable Line Spacing Gratings, using the stationary phase method:  
N.Canestrari, V.Bisogni, A.Walter, Y.Zhu, J.Dvorak, E.Vescovo, O.Chubar, Proc. SPIE Vol. 9209, 92090I (2014)

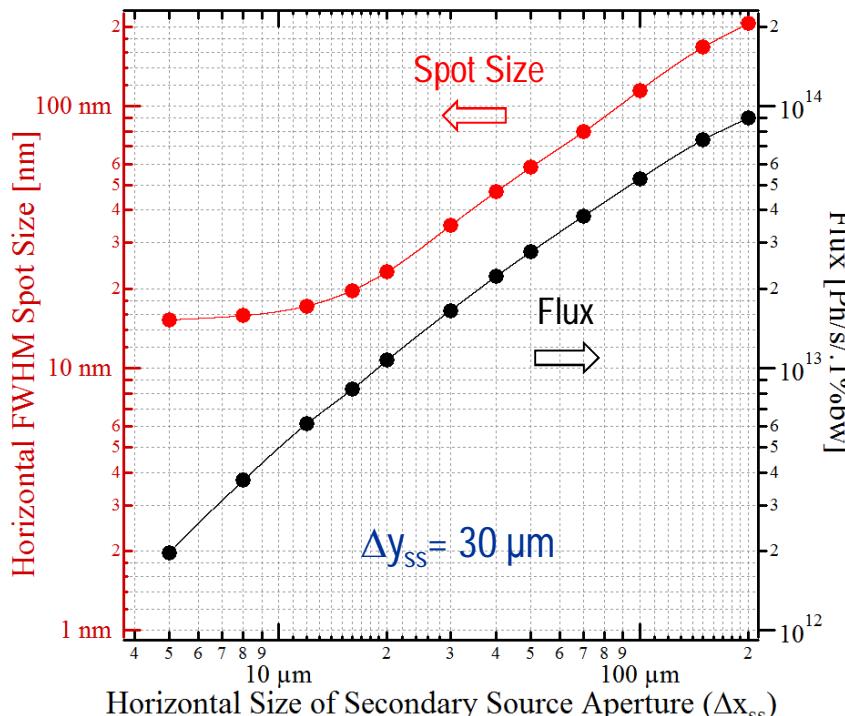
R&D Direction: Improvement of efficiency and reliability of Partially-Coherent "Forward" Simulation

# NSLS-II Hard X-Ray Nanoprobe (HXN) Beamline Optical Scheme and Wavefront Propagation Simulation

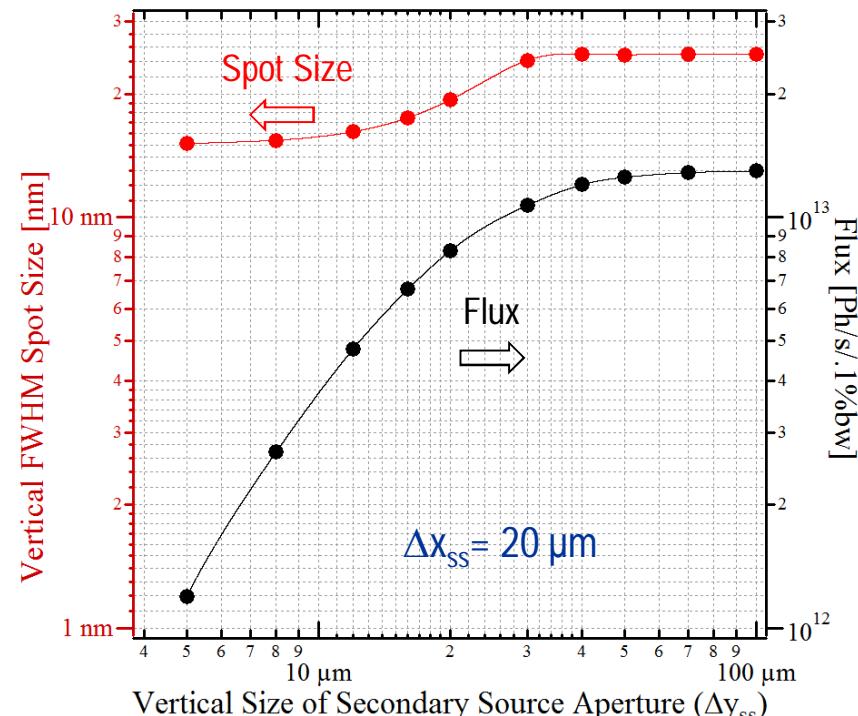


# Final Focal Spot Size and Flux at Sample vs Secondary Source Aperture Size (HXN, NSLS-II)

Horizontal Spot Size and Flux  
vs Horizontal Secondary Source Aperture Size



Vertical Spot Size and Flux  
vs Vertical Secondary Source Aperture Size



Secondary Source Aperture located at 94 m from Undulator

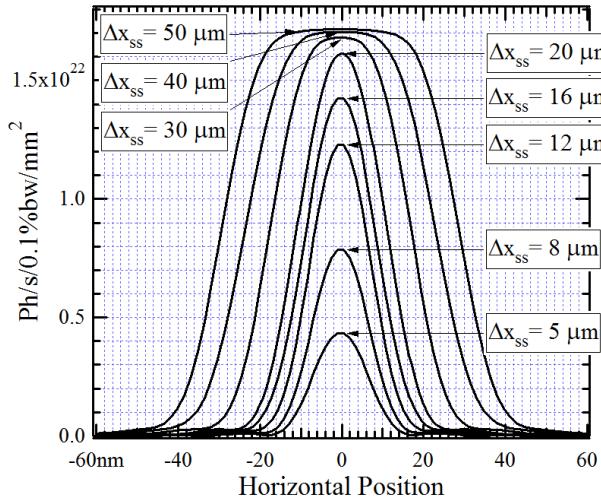
Spot Size and Flux calculated for Nanofocusing Optics simulated by Ideal Lens  
with  $F = 18.14$  mm,  $D = 150$  μm located at 15 m from Secondary Source (109 m from Undulator)

Pan-Am SRI-2010

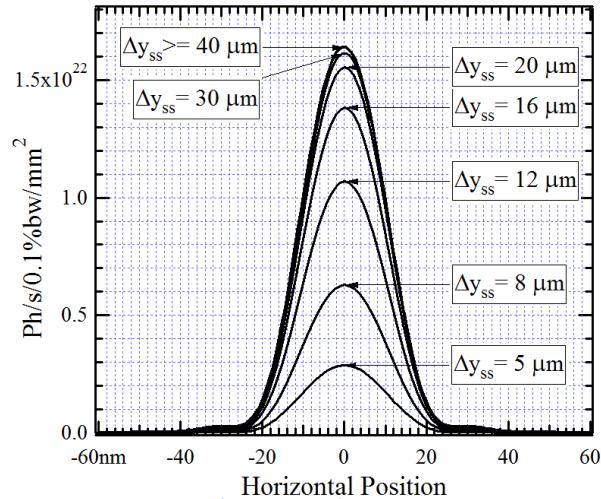
# Intensity Distributions at Sample for Different Secondary Source Aperture Sizes at HXN (NSLS-II)

## In Horizontal Median Plane ( $y = 0$ )

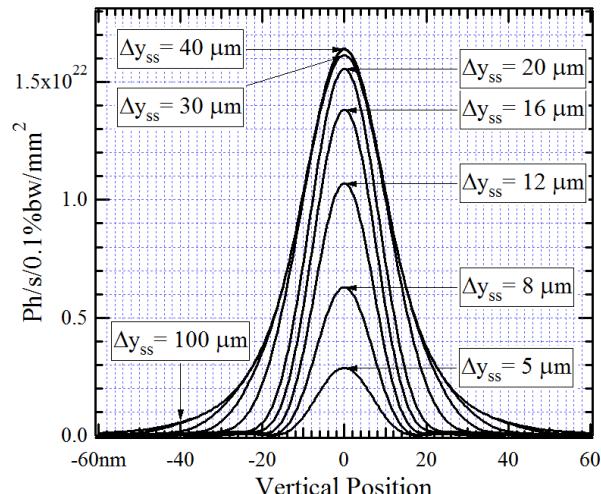
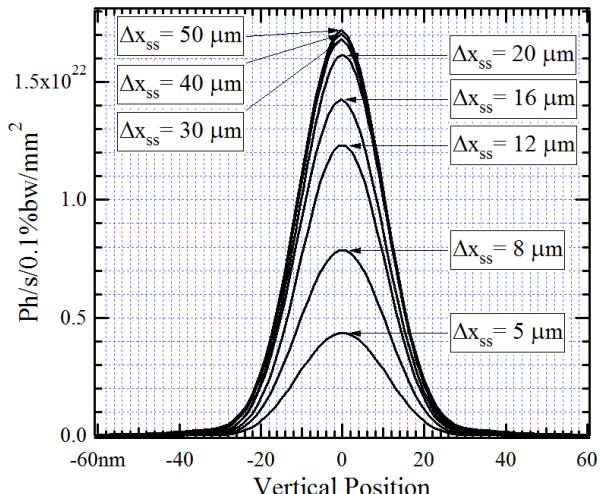
For Different Horizontal SSA Sizes ( $\Delta x_{ss}$ )



For Different Vertical SSA Sizes ( $\Delta y_{ss}$ )

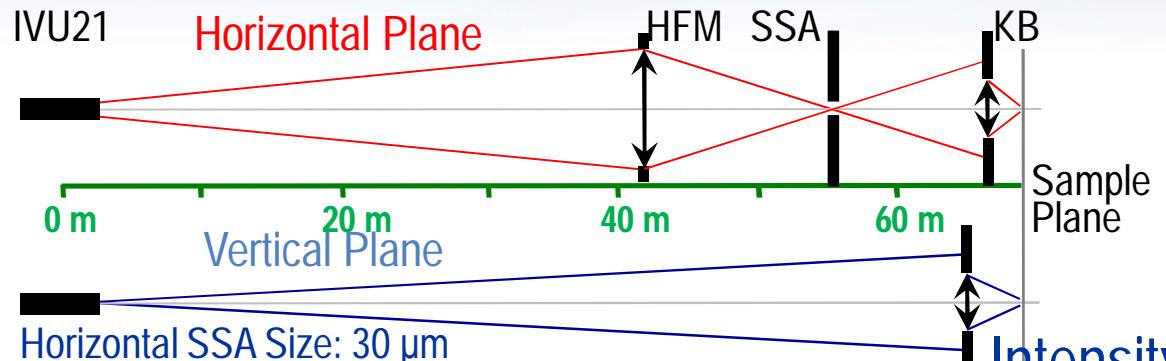


## In Vertical Median Plane ( $x = 0$ )

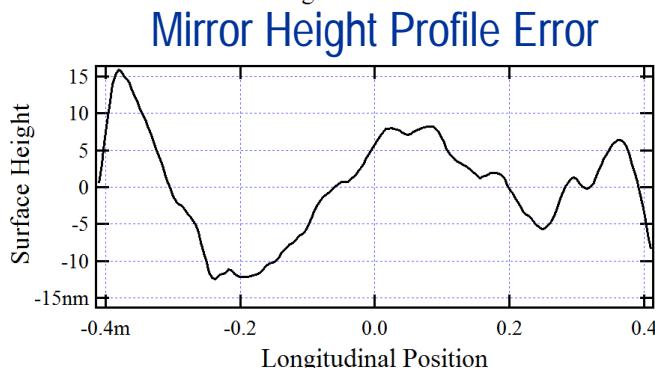
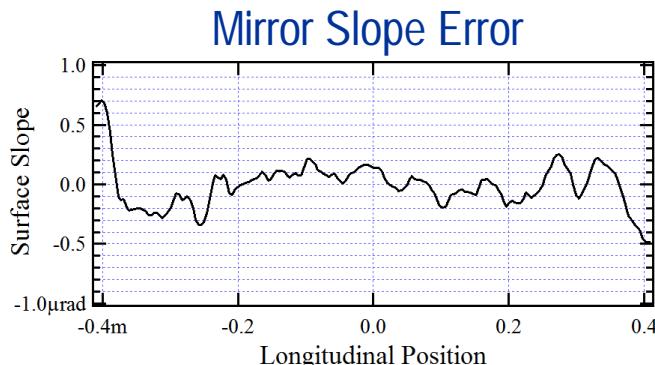


For Nanofocusing Optics with  $F = 18.14 \text{ mm}$ ,  $D = 150 \mu\text{m}$  ( $\Delta r \approx 15 \text{ nm}$ ;  $E_{ph} \approx 10 \text{ keV}$ )  
SSA located at 94 m, Nanofocusing Optics at 109 m from Undulator

# Partially-Coherent Wavefront Propagation Simulations for a Beamline with Grazing-Incidence Focusing Mirrors, Taking Into Account Their Imperfections (FMX @ NSLS-II)

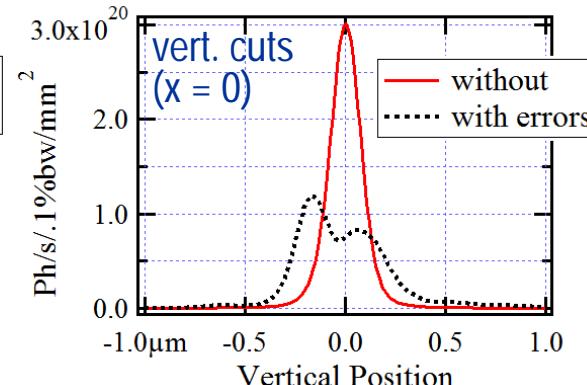
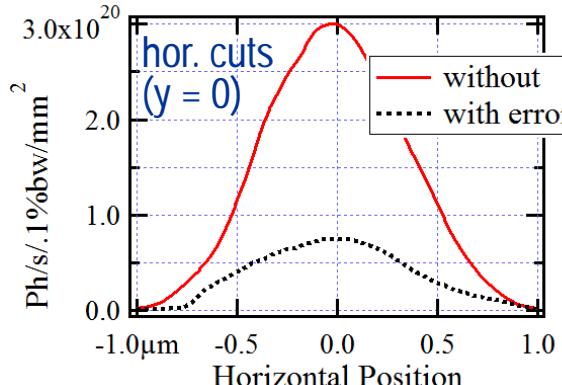
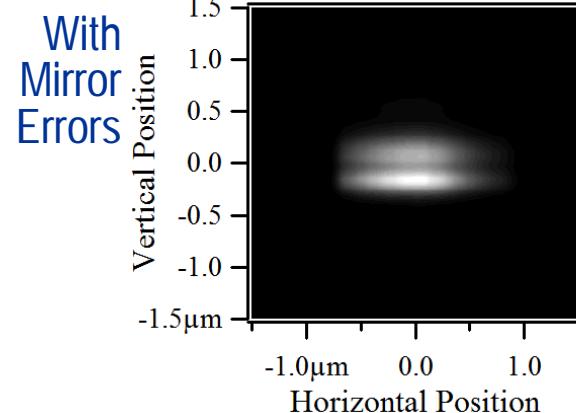
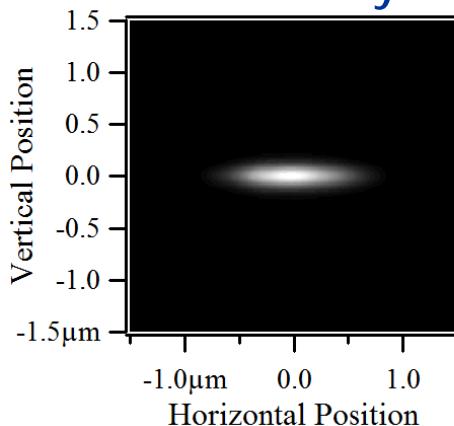


Horizontal SSA Size: 30  $\mu\text{m}$   
 Photon Energy: 12.7 keV  
 Flux at Sample:  $\sim 5.4 \times 10^{13} \text{ ph/s/.1\%bw}$

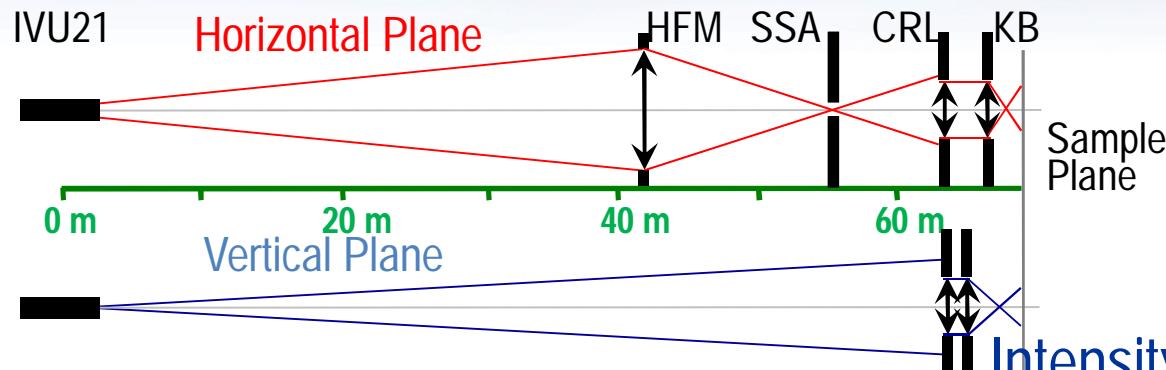


KB simulated using Grazing-Incidence "Thick Optical Element" Propagator based on "Local Ray-Tracing".  
 KB Surface Height Error simulated by corresponding Phase Shifts ("Masks") in Transverse Plane at Mirror Locations.

## Intensity Distributions at Sample



# Using CRL for Producing “Large Spot” at Sample of FMX Beamline @ NSLS-II



Horizontal SSA Size: 30  $\mu\text{m}$   
Photon Energy: 12.7 keV

## CRL “Transfocator”:

8 Horizontally + 3 Vertically-Focusing Be Lenses

$R_{\min} = 200 \mu\text{m}$

$F_h \approx 5.9 \text{ m}$ ,  $F_v \approx 15.8 \text{ m}$

Geom. Ap.: 1 mm x 1 mm

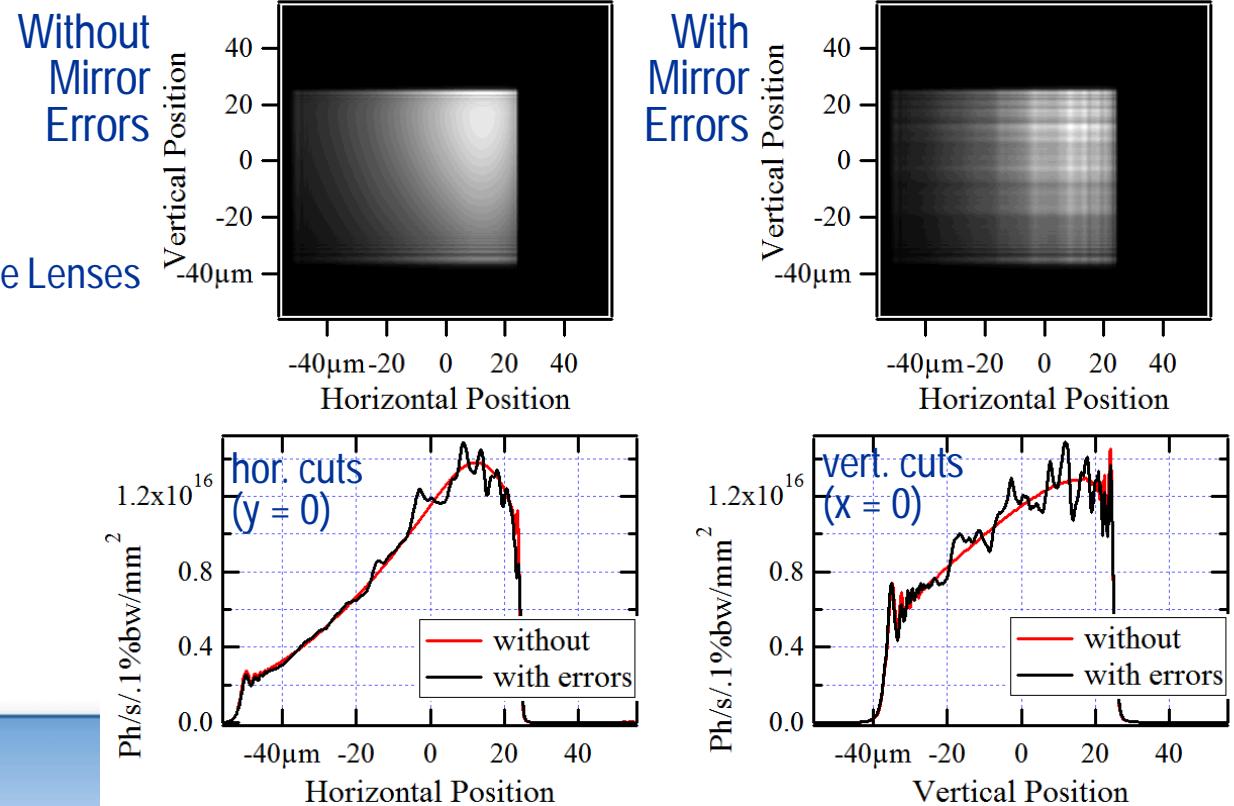
Located at 0.75 m before VKB edge  
(10 m after SSA)

Flux Losses at CRL: ~1.6 times

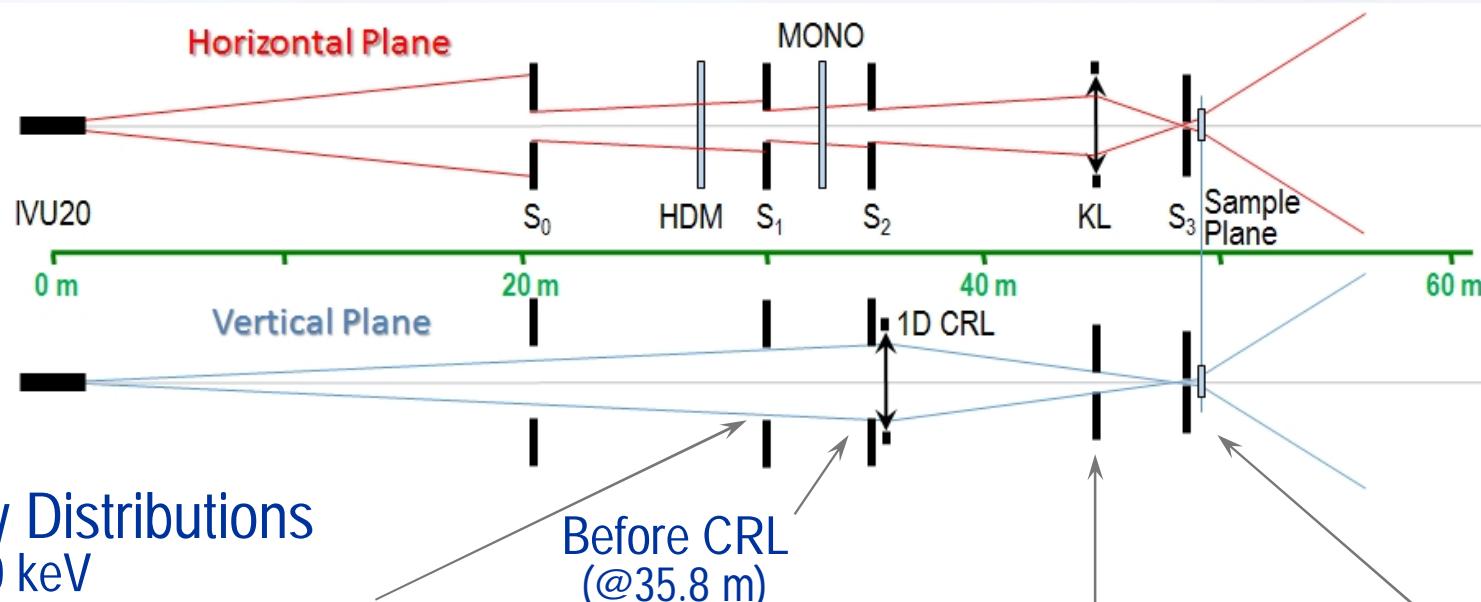
## Source:

Electron Current: 0.5 A  
Horizontal Emittance: 0.55 nm (“ultimate”)  
Vertical Emittance: 8 pm  
Undulator: IVU21-1.5 m centered at +1.25 m  
from Low-Beta Straight Section Center

## Intensity Distributions at Sample



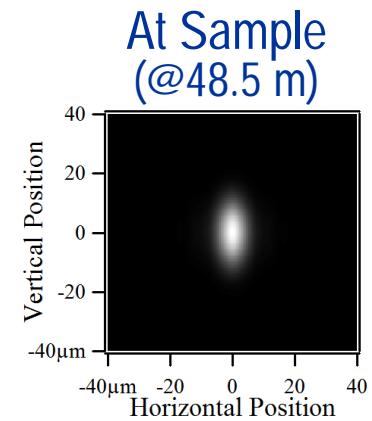
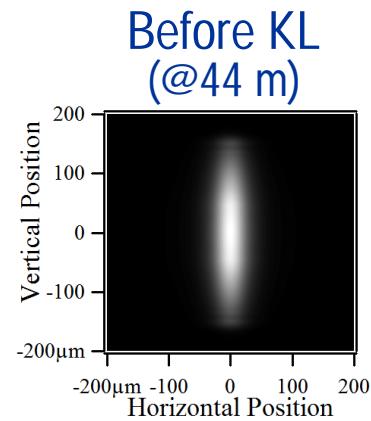
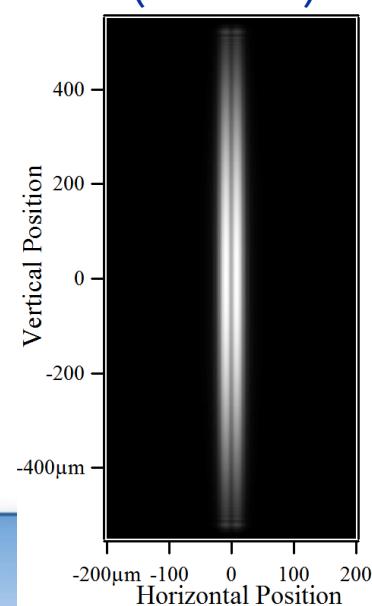
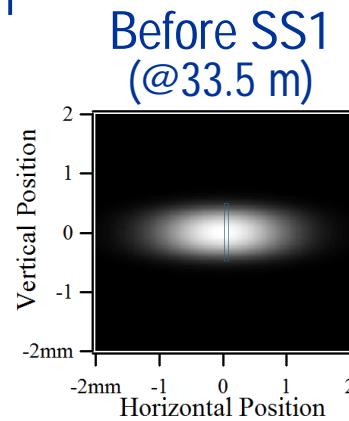
# Partially-Coherent Wavefront Propagation Simulations for CHX Beamline @ NSLS-II



Intensity Distributions

for E = 10 keV

$$\Delta S_{1x} = 44 \mu\text{m}$$
$$\Delta S_{1y} = 1 \text{ mm}$$



Flux:  $10^{13} \text{ ph/s/.1%bw}$



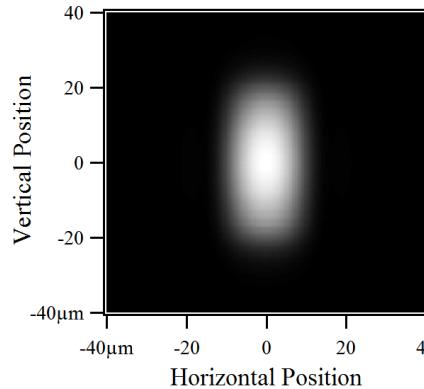
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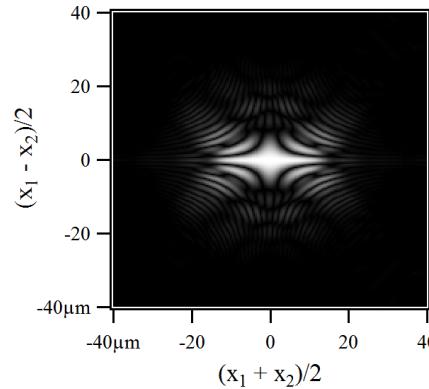
National Synchrotron  
Light Source II

# Tracking Intensity and Degree of Transverse Coherence at a Sample (CHX @ NSLS-II)

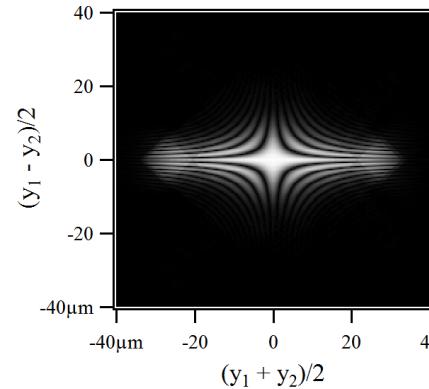
Intensity Distribution



Degree of Transverse Coherence  
In Horizontal Mid-Plane

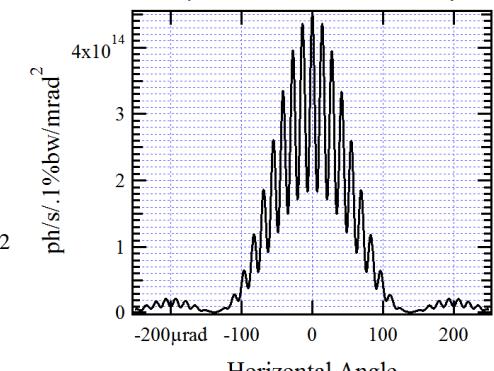


In Vertical Mid-Plane

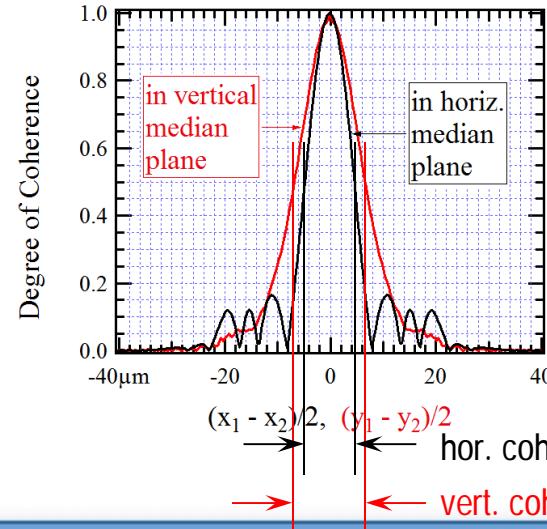
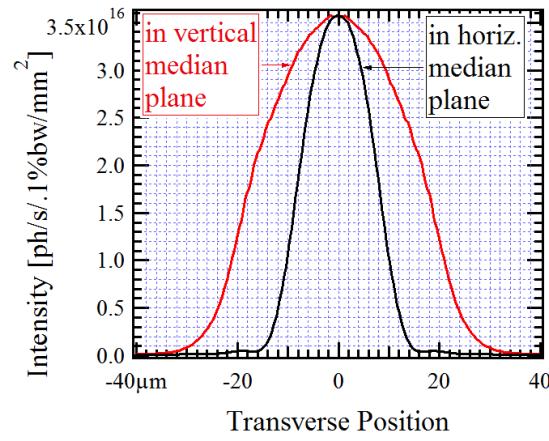
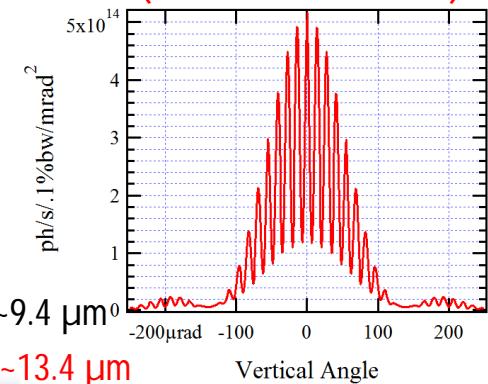


Angular Intensity (far field)  
after Two Slits  
separated by 10 μm

In Horizontal Plane  
(after vertical slits)



In Vertical Plane  
(after horizontal slits)



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Good agreement with 2-slit interference simulation results

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# Partially-Coherent Wavefront Propagation Simulations for Inelastic X-ray Scattering Beamlne with Advanced High-Resolution Crystal Optics (IXS @ NSLS-II)

A.Suvorov, Y.Q.Cai, J.P.Sutter, O.Chubar, Proc. of SPIE Vol. 9209, 92090H (2014)

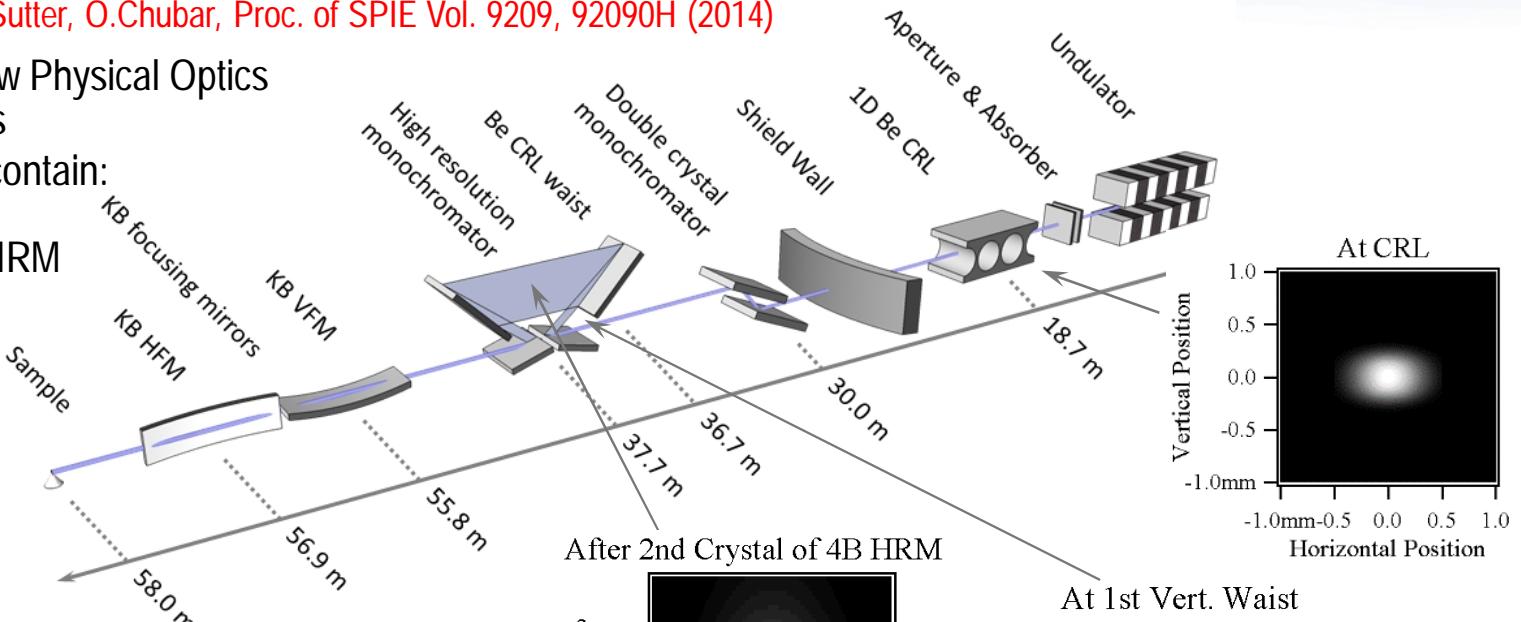
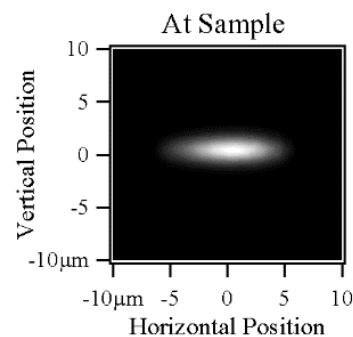
Extended testing of new Physical Optics

Propagator for Crystals

IXS Monochromators contain:

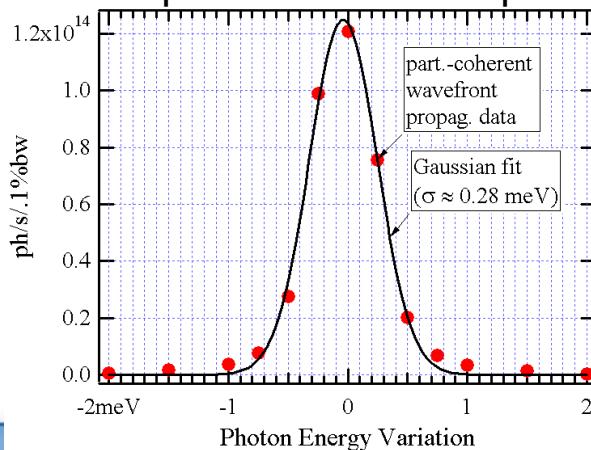
DCM: 2 Crystals

HRM: 4 Crystals of HRM



Mirror Surface  
Error is not taken  
into account

$$E_0 \approx 9131.7 \text{ eV}$$



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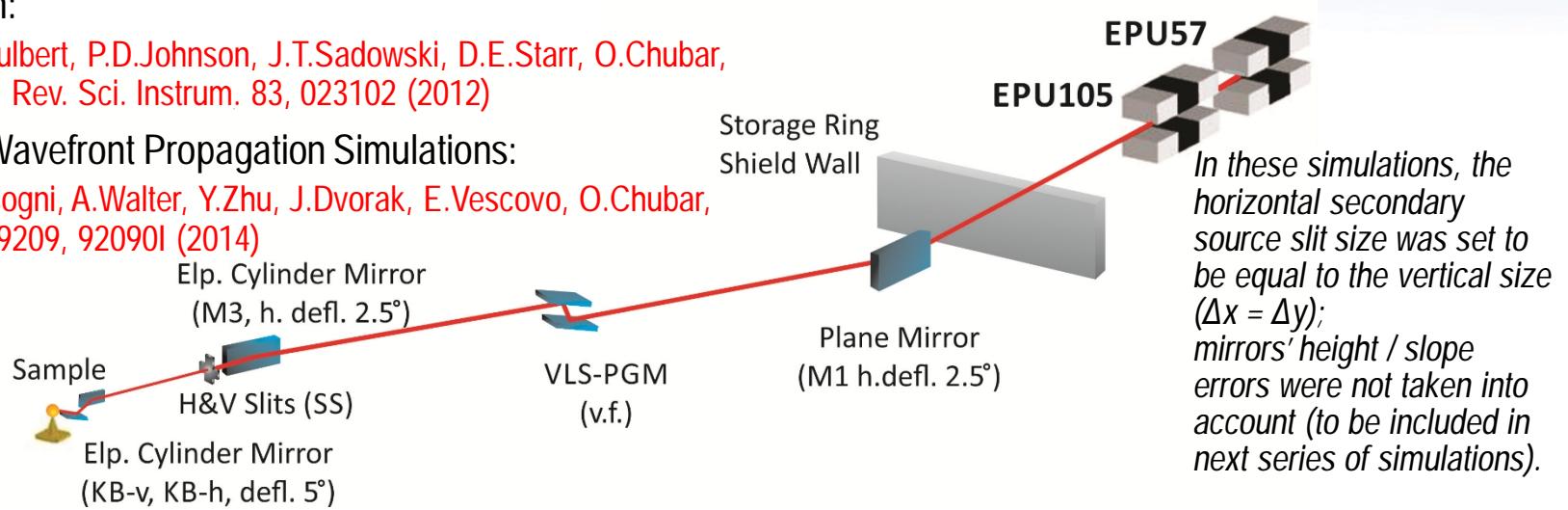
# Partially-Coherent Wavefront Propagation Simulations for a Soft X-ray Beamlne with VLS grating (ESM @ NSLS-II)

Beamlne Design:

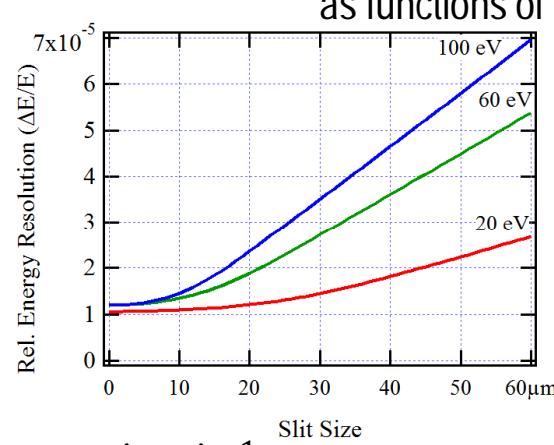
R.Reininger, S.L.Hulbert, P.D.Johnson, J.T.Sadowski, D.E.Starr, O.Chubar,  
T.Valla, E.Vescovo, Rev. Sci. Instrum. 83, 023102 (2012)

Part.-Coherent Wavefront Propagation Simulations:

N.Canestrari, V.Bisogni, A.Walter, Y.Zhu, J.Dvorak, E.Vescovo, O.Chubar,  
Proc. of SPIE Vol. 9209, 920901 (2014)

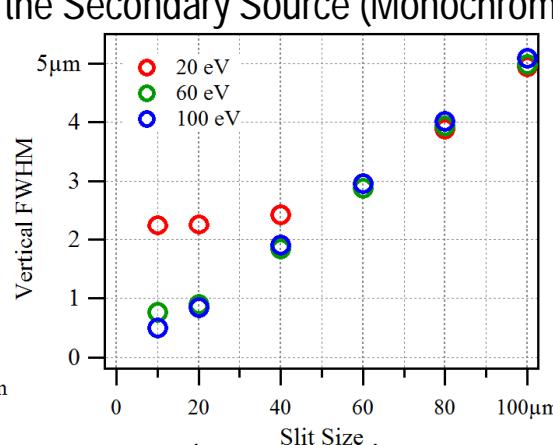


Energy Resolution



$$\Delta E/E > (mN)^{-1}$$

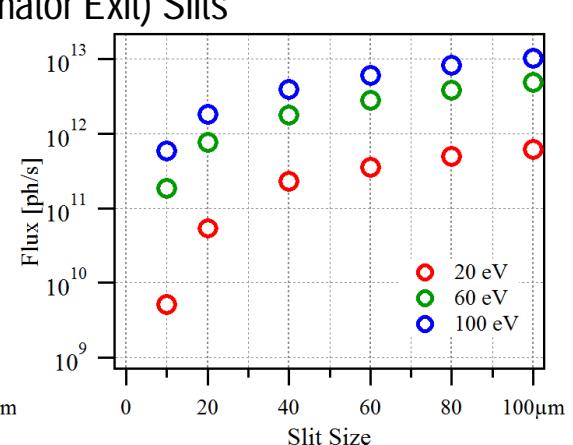
Spatial Resolution



Two different VLS Gratings (160 mm long) were used:

$a_0 = 800$  lines/mm for  $E = 20$  eV;  $a_0 = 600$  lines/mm for  $E = 60, 100$  eV

Flux (finite-bandwidth) at Sample

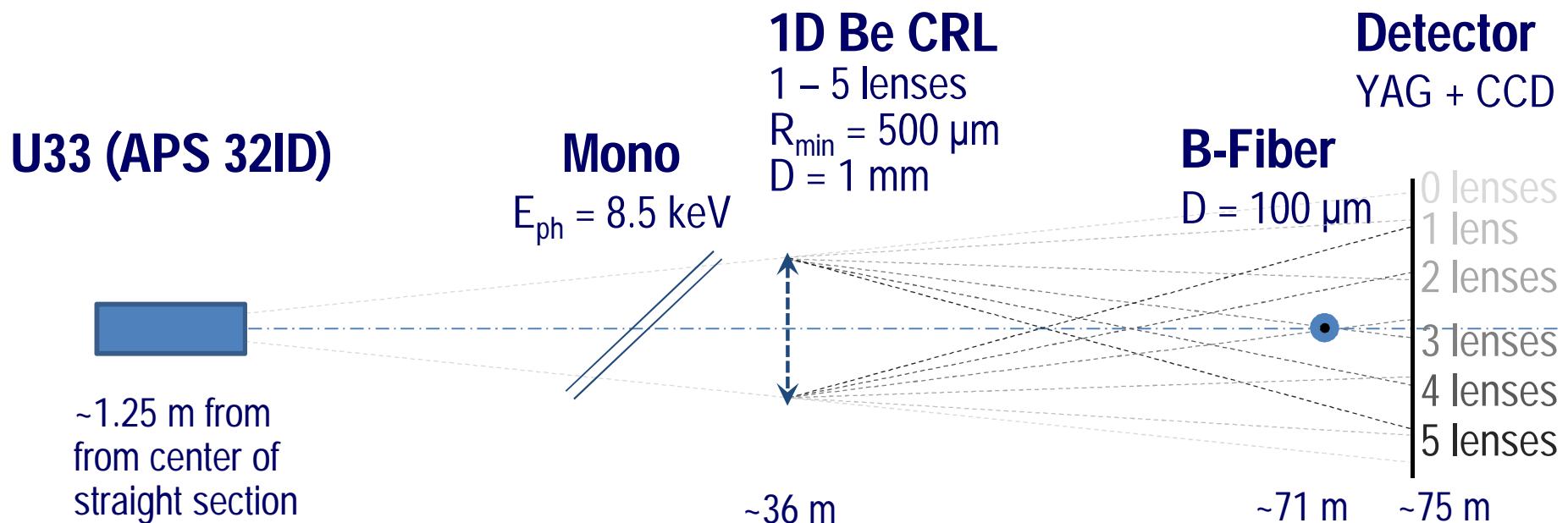


# Approach to Coherence Preservation Diagnostics

## Assisted by Simulations (Illustration)

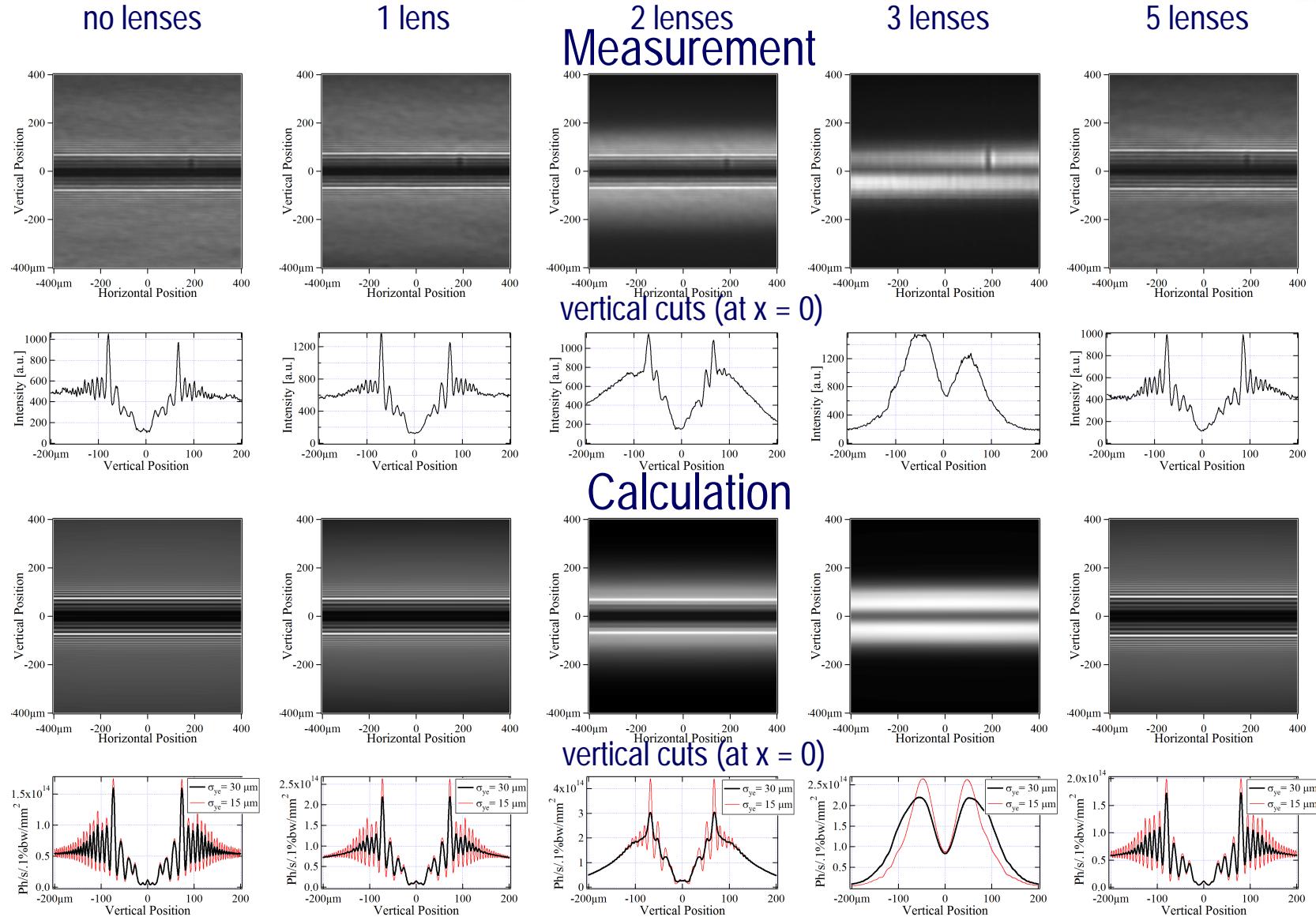
- V.Kohn, I.Snigireva and A.Snigirev, "Direct measurement of transverse coherence length of hard X-rays from interference fringes", Phys. Rev. Lett., vol.85(13), p.2745 (2000)
- A.Snigirev, V.Kohn, I.Snigireva, B.Lengeler, "A compound refractive lens for focusing high-energy X-rays", Nature, vol.384, p.49 (1996)
- O.Chubar, A.Fluerasu, Y.S.Chi, L.Berman, L.Wiegart, W.-K.Lee, J.Baltser, "Experimental characterization of X-ray transverse coherence in the presence of beam transport optics", J. Phys.: Conf. Ser. 425, 052028 (2013)

Optical scheme of test experiments with CRL and a Boron fiber probe



# Intensity Distributions in the B-fiber Based Interference Scheme for Different Numbers of CRL in Optical Path

Simulations allow to conclude about coherence preservation in presence of any beamline optics!

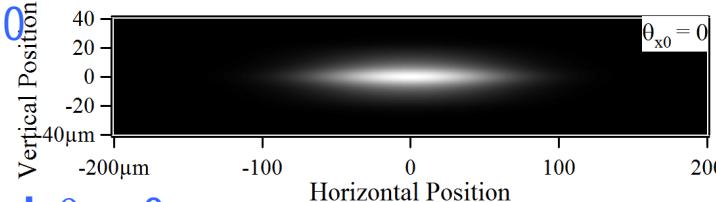


# Intensity Distributions of Focused Wiggler Radiation from Partially-Coherent Wavefront Propagation Calculations

On-Axis Collection:  $\theta_{x0} = 0$ ,  $\theta_{y0} = 0$

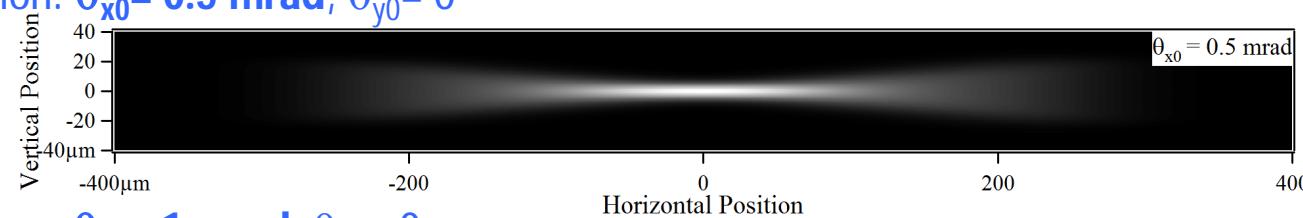
$$|\theta_x - \theta_{x0}| < 0.1 \text{ mrad}$$

$$|\theta_y - \theta_{y0}| < 0.1 \text{ mrad}$$

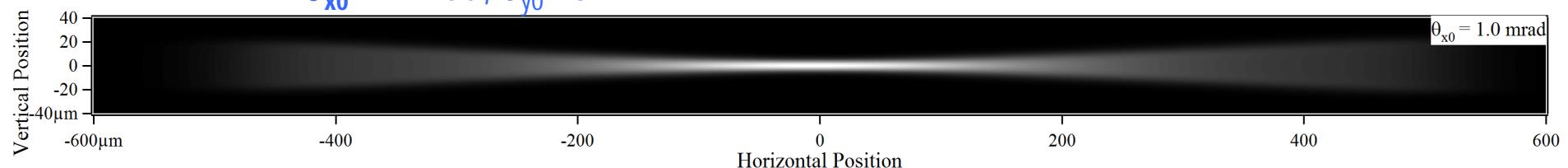


1 : 1 Imaging Scheme  
with "Ideal Lens"

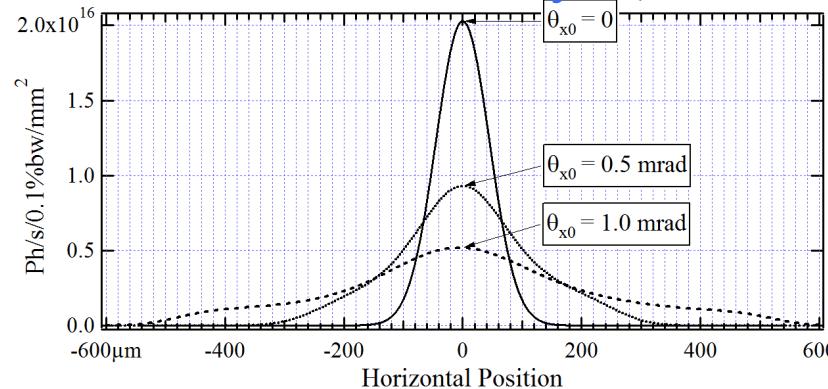
Off-Axis Collection:  $\theta_{x0} = 0.5 \text{ mrad}$ ,  $\theta_{y0} = 0$



$$\theta_{x0} = 1 \text{ mrad}, \theta_{y0} = 0$$

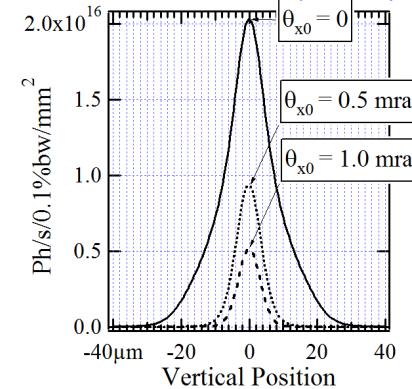


Horizontal Cuts ( $y = 0$ )



NSLS-II Low-Beta Straight Section  
 $I = 0.5 \text{ A}$ ,  $\varepsilon_x = 0.9 \text{ nm}$ ,  $\varepsilon_y = 8 \text{ pm}$

Vertical Cuts ( $x = 0$ )



SCW40:  $\lambda_u = 40 \text{ mm}$ ,  $B_{\max} = 3 \text{ T}$ ,  $L = 1 \text{ m}$   
Photon Energy:  $E_{\text{ph}} = 10 \text{ keV}$

# Intensity Distributions of Monochromatic Radiation from ESRF-U 2PW in 1:1 Imaging Plane

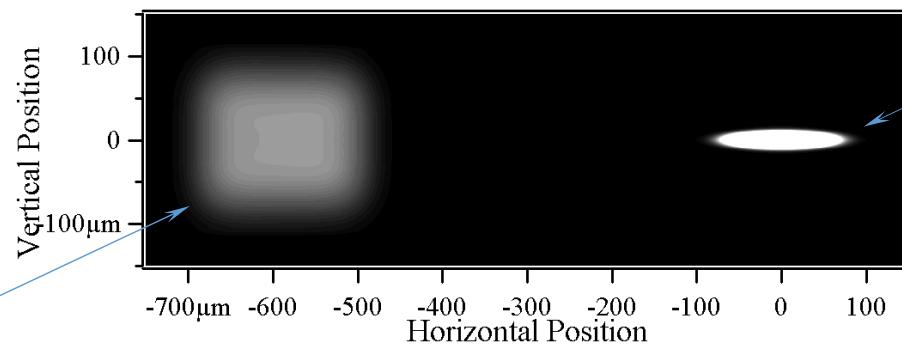
"Non-saturated"  
Image Plot:



Focusing by Ideal Lens  
located at:  $R = 30$  m  
Lens Aperture:  
 $\Delta x = 8$  mm,  $\Delta y = 10$  mm  
Photon Energy: 5 keV

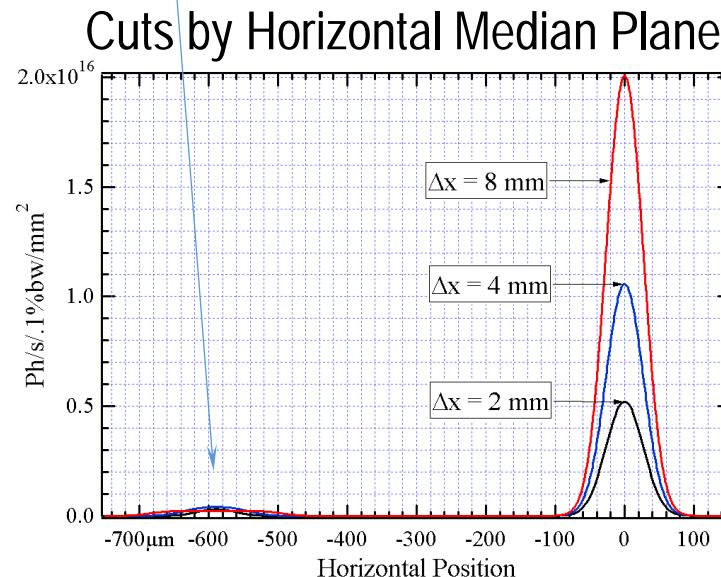
"Saturated"  
Image Plot:

(max. intensity 50 times  
lower than in the "non-  
saturated" plot)

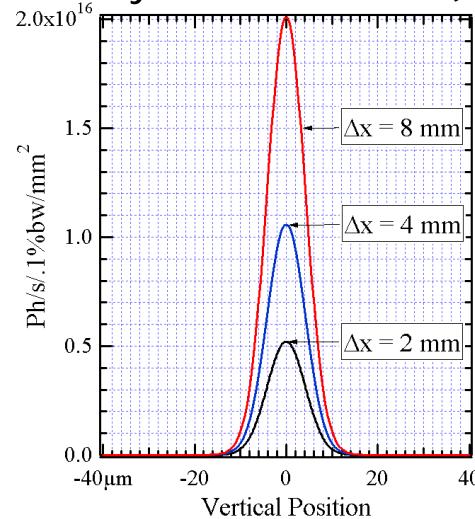


From 2PW  
(well focused)

From Downstream  
Dipole (out of focus)



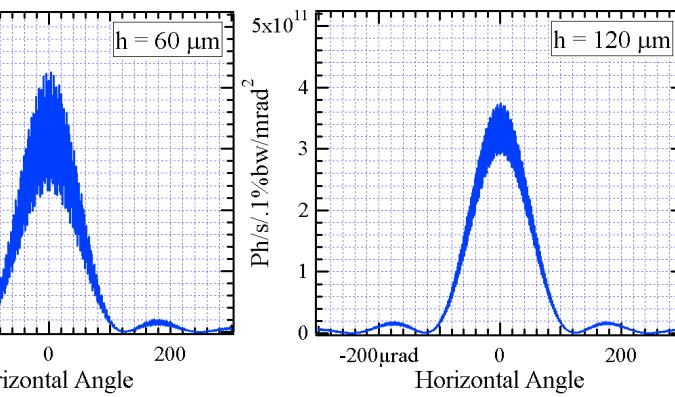
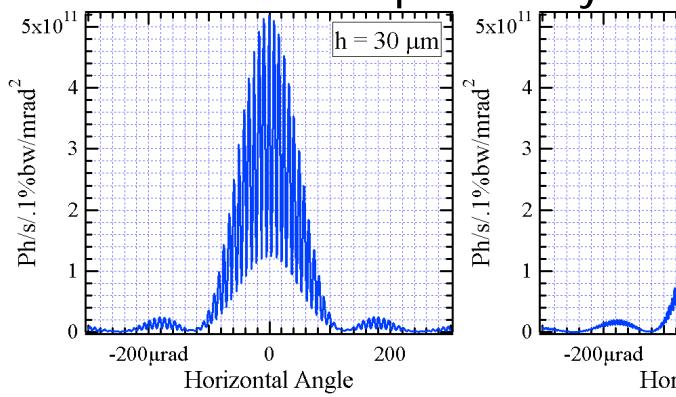
Cuts by Vertical Plane ( $x = 0$ )



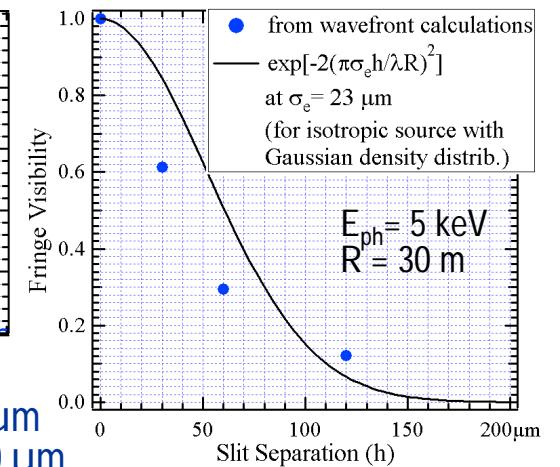
at Different  
Horizontal  
Apertures

# Estimating Degree of Coherence (/ Transverse Coherence Lengths) of Radiation from ESRF-U 2PW by Simulating Young's 2-Slit Interference Schemes

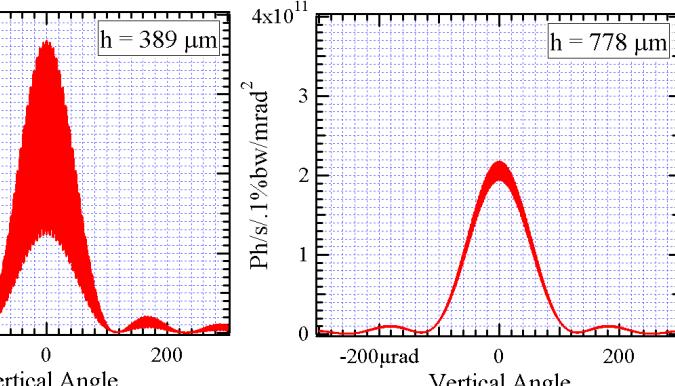
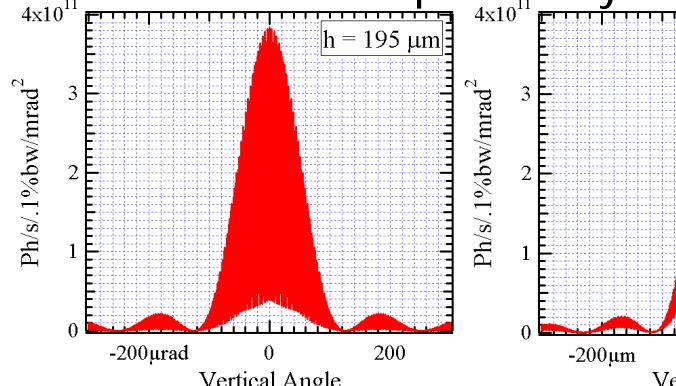
Far-Field Interference Patterns from 2 Vertical Slits Separated by Horizontal Distance  $h$



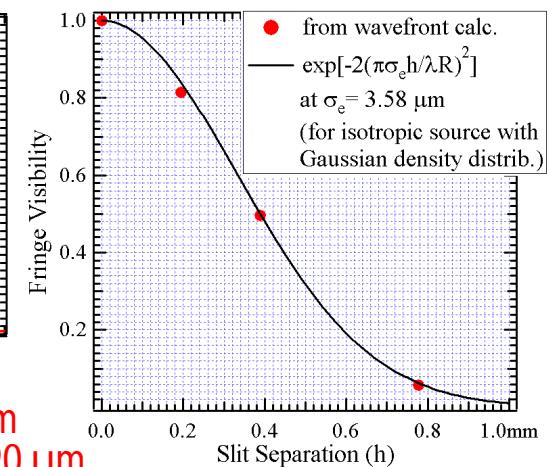
Fringe Visibility vs  $h$  in Horizontal Plane



Far-Field Interference Patterns from 2 Horizontal Slits Separated by Vertical Distance  $h$



Fringe Visibility vs  $h$  in Vertical Plane



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